Saturation: Colour Glass Condensate and colour sources *

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In the recent years we have seen a lot of activity around systems and experiments, like DIS at HERA or the heavy-ion experiments at RHIC, involving a large number of partons due to the high enery and/or the high number of participants of those experiments. The main problem in this regime is that of the high parton densities. In fact, in most of the models of multiparticle production, two contributions to the multiplicity are considered: one proportional to the number of participant nucleons, N_{part}, and a second one proportional to the number of collisions, $N_{part}^{4/3}$. In order to get the right multiplicities at RHIC it is necessary to lower the second contribution. A possible mechanism for this is the saturation. Here, I am going to review the saturation of parton densities in the initial state, in two different frameworks: the Colour Glass Condensate and the string clustering.

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1. THE COLOUR GLASS CONDENSATE

1.1. Parton saturation at small x and the saturation momentum

It has been much activity in the last years trying to understand the physics of nuclear and hadronic processes in the regime of very small Bjorken's x (very high energy). The main problem in this regime is that of the high parton densities. At high energy, the QCD cross sections are controlled by small longitudinal momentum gluons in the hadron wavefunction, whose density grows rapidly with increasing energy or decreasing x, due to the enhancement of radiative process. If one applies perturbation theory to this regime, one finds that, by resumming dominant radiative corrections at high energy, the BFKL equation leads to a gluon density that grows like a power

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of s and in consequence to a cross section that violates the Froissart bound. Nevertheless, the use of perturbation theory to high-energy problems is not obvious. In fact, the BFKL and DGLAP equations are linear equations that neglet the interaction among the gluons. With increasing energy, recombination effects –that are non-linear– favored by the high density of gluons should become more important and lead to an eventual *saturation* of parton densities.

These effects become important when the interaction probability for the gluons becomes of order one. Taking $\frac{\alpha_s N_c}{Q^2}$ as the transverse size of the gluon and $\frac{xG(x,Q^2)}{\pi R^2}$ as the density of gluons, the interaction probability is expressed by

$$\frac{\alpha_s N_c}{Q^2} \times \frac{xG(x,Q^2)}{\pi R^2} . \tag{1}$$

Equivalently, for a given energy, saturation occurs for those gluons having a sufficiently large transverse size $r_\perp^2 \sim 1/Q^2$, larger than some critical value $1/Q_s(x,A)$. So the phenomenon of saturation introduces a characteristic momentum scale, the *saturation momentum* $Q_s(x,A)$, which is a measure of the density of the saturated gluons, and grows rapidly with $1/x$ and A (the atomic number). The probability of interaction –that can be understood as "overlapping" of the gluons in the transverse space– becomes of order one for those gluons with momenta $Q^2 \lesssim Q_s(x,A)$ where

$$Q_s^2(x,A) = \alpha_s N_c \, \frac{xG(x,Q_s^2)}{\pi R^2} \equiv \frac{(\text{colour charge})^2}{\text{area}} . \tag{2}$$

For $Q^2 \lesssim Q_s^2(x,A)$, the non-linear effects are essential, since they are expected to soften the growth of the gluon distribution with $\tau \equiv \ln(1/x)$. For a nucleus, $xG_A(x,Q_s^2) \propto A$ and $\pi R_A^2 \propto A^{2/3}$, so eq. (2) predicts $Q_s^2 \propto A^{1/3}$. One can estimate the saturation scale by inserting the BFKL approximation into eq. (2). This gives (with $\delta \approx 1/3$ and $\lambda \approx c\bar{\alpha}_s$ in a first approximation):

$$Q_s^2(x,A) \sim A^\delta \, x^{-\lambda} , \qquad c = [-\beta + \sqrt{\beta(\beta + 8\omega)}\,]/2 = 4.84... , \tag{3}$$

which indicates that an efficient way to create a high-density environment is to combine large nucleai with moderately small values of x, as it is done at RHIC.

This equation also shows that for sufficiently large energy or x small enough, $Q_s^2(x,A) \gg \Lambda_{QCD}^2$ and $\alpha_s(Q_s) \ll 1$, which characterizes the regime of weak coupling QCD. But although the coupling is small, the effects of the interactions are amplified by the large gluon density: at saturation, $G_A(x,Q_s^2) \sim 1/\alpha_s(Q_s) \gg 1$, so the gluon modes have large occupation numbers, of order $1/\alpha_s$ (corresponding to strong classical fields $A \sim 1/g$), which suggests the use of semi-classical methods.

1.2. The effective theory for the CGC: the Renormalization Group Equation.

One can write a classical effective theory based on this general idea: the fast partons -valence quarks with large longitudinal momentum- are considered as a *classical source* that emits soft gluons -with smaller longitudinal momenta- which are treated as *classical colour fields* $A[\rho]$. The fast partons move nearly at the speed of light in the positive x^+ direction, and generate a colour current $J^\mu = \delta^{\mu +}\rho$. By Lorentz contraction, the support of the charge density ρ is concentrated near the light-cone longitudinal coordinate $x^- = 0$. By Lorentz time dilation, ρ is independent of the light-cone time x^+.

The Yang Mills equation describing the soft gluon dynamics reads

$$D_\nu F^{\nu\mu} \ = \ \delta^{\mu +}\rho(x^-, \mathbf{x}) \ . \tag{4}$$

Physical quantities, as the unintegrated gluon distribution, are obtained as an average over ρ:

$$< A^i(X)\, A^i(Y) >_x = \int D[\rho]\ W_x[\rho]\ A^i[\rho](X)\, A^i[\rho](Y)\ , \tag{5}$$

where $A^i(X)$ corresponds to the classical solution for a given ρ, and $W_x[\rho]$ is a gauge-invariant weight function for ρ. What we are doing is a kind of Born-Oppenhaimer approximation: first, we study the dynamics of the classical fields (Weizsacher-William fields) for a given configuration ρ of the colour charges, and second, we average over all possible configurations.

For the classical solution we find:

$$F^{+i}(x^-, x_\perp) = \delta(x^-)\frac{i}{g}V(x_\perp)(\partial^i V(x_\perp)^\dagger) = \partial^+ A^i\ , \tag{6}$$

where $V(x_\perp)$ is the Wilson line

$$V^\dagger(x_\perp) \equiv \mathrm{Pexp}\left\{ig\int dx^- A^+(x^-, \mathbf{x})\right\} \tag{7}$$

and $A^+[\rho]$ is the solution of the equation of motion (4) in the covariante gauge: $-\nabla_\perp^2 A^+ = \rho$.

The weight function $W_\tau[\rho]$ is obtained by integrating out the fast partons, so it depends upon the rapidity scale $\tau = \ln(1/x)$ at which one considers the effective theory. This can be taken into account via a one-loop background field calculation, and leads to a renormalization group equation (REG) for $W_\tau[\rho]$ which shows how the correlations of ρ change with increasing τ. Schematically:

$$\frac{\partial W_\tau[\rho]}{\partial \tau} \ = \ \frac{1}{2}\int_{x_\perp, y_\perp} \frac{\delta}{\delta \rho_x^a}\ \chi_{xy}^{ab}[\rho]\ \frac{\delta}{\delta \rho_y^b}\ W_\tau[\rho]\ . \tag{8}$$

This is a functional diffusion equation, where the kernel $\chi[\rho]$ plays the role of the diffusion coefficient in the functional space spanned by $\rho(x^-, x_\perp)$. This kernel is positive definite and non-linear in ρ to all orders. It depends upon ρ via the Wilson line (7).

As it has been said above, the physical quantities, as the gluon density, are obtained as an average over ρ:

$$n(x, k_\perp) \equiv \frac{1}{\pi R^2} \frac{\mathrm{d}N}{\mathrm{d}\tau\, \mathrm{d}^2 k_\perp} \propto \langle F^{+i}(k_\perp) F^{+i}(-k_\perp) \rangle_x \ . \tag{9}$$

In order to calculate this average, we need to solve the REG (8). Approximate solutions to this equation can be obtained in two limiting cases:

• At low energy, or large transverse momenta $k_\perp^2 \gg Q_s^2(x)$, we are in a dilute regime where fields and sources are weak, and the Wilson lines can be expanded to lowest order in A^+, $V^\dagger(x_\perp) \approx 1 + ig \int dx^- A^+(x^-, x_\perp)$. In this case, the REG equation reduces to the BFKL equation, and the gluon density of eq. (9) grows both with $1/k_\perp^2$ and $1/x$ (Bremsstrahlung): $n(x, k_\perp) \sim \frac{1}{k_\perp^2} \frac{1}{x^{\omega \alpha_s}}$.

• At high energies, or low momenta $k_\perp^2 \lesssim Q_s^2(x)$, the colour fields are strong, $A^+ \sim 1/g$, so the Wilson lines rapidly oscillate and average away to zero: $V \approx V^\dagger \approx 0$. Then the kernel χ becomes independent of ρ, and we obtained a gluon density that increases linearly with the evolution "time" $\tau = ln(1/x)$: $n(x, k_\perp) \sim \frac{1}{\alpha_s} \ln \frac{Q_s^2(x)}{k_\perp^2} \propto \ln \frac{1}{x}$. That is, we find *saturation* for the gluon density, that grows logaritmically with the energy since $\tau \sim \ln s$: unitarity is restored.

We call the high density gluonic matter at small-x described by this effective theory a *Colour Glass Condensate* (CGC) [1]: *Colour* since gluons carry colour under $SU(N_c)$; *Glass* since we have a random distribution of time-independent colour charges which is averaged over in the calculation of physical observables, in order to have a gauge independent formulation; and *Condensate* because at saturation the gluon density is of order $1/\alpha_s$, typical of condensates, so we have a system of saturate gluons that is a Bose condensate.

1.3. Phenomenology at RHIC

The particle production in RHIC collisions has been analyzed from the perspective of the CGC, considering it as a pertinent description of the initial conditions. Taking into account that the multiplicity is proportional to the number of gluons -parton-hadron duality-, the centrality dependence of multiparticle production has been related to the density of gluons [2],

that at saturation (see eq. (2)):

$$\frac{\mathrm{d}N}{\mathrm{d}y} \sim xG(x,Q_s^2) = \frac{\pi R_A^2\, Q_s^2(x,A)}{\alpha_s(Q_s^2)} \tag{10}$$

where $\pi R_A^2 \propto N_{part}^{2/3}$ corresponds to the nuclear overlap area, and Q_s is the saturation momentum for the considered centrality, $Q_s^2(x,A) \propto N_{part}^{1/3}$. To compute the centrality dependence, it is necessary to know the evolution of the gluon structure function, wich is governed by the DGLAP equation. Taking $1/\alpha_s(Q_s^2) \approx \ln(Q_s^2/\Lambda_{QCD}^2) \sim \ln N_{part}$, one finally finds that the multiplicity per participant behaves as $\ln N_{part}$.

Besides, it has been obtained from saturation a proportionality between the mean transverse momentum and the multiplicities,

$$< p_T >^2 \sim \frac{1}{\pi R_A^2}\frac{\mathrm{d}N}{\mathrm{d}y} \ . \tag{11}$$

This observation indicates that the p_T broadening seen in elementary and heavy ion collisions results from the same physics, the intrinsic generated p_T broadening in the partonic phase [3].

2. STRING CLUSTERING

In the clustering approach, the colour strings created in the nuclear collisions are considered as effective sources with a fixed transverse area, $r_\perp \approx 0.2$ fm. Notice that this radius coincides with the estimate saturation momentum of the CGC at RHIC. If the string density is high enough, the strings *overlap*, forming clusters [4], very much like disks in continuum two-dimensional percolation theory. In order to calculate the physical observables, as the particle multiplicity or the mean transverse momentum, we need to study the dynamics of those clusters.

We assume that a cluster of n strings behaves as a single string with a higher colour field $\vec{Q}_n$, corresponding to the vectorial sum of the colour charges of each individual $\vec{Q}_1$ string. The resulting colour field covers the area S_n of the cluster. As $\vec{Q}_n^2 = (\sum_1^n \vec{Q}_1)^2$, and the individual string colours may be oriented in an arbitrary manner, the average $\vec{Q}_{1i}\vec{Q}_{1j}$ is zero, so $\vec{Q}_n^2 = n\vec{Q}_1^2$. $\vec{Q}_n$ depends also on the area S_1 of each individual string that comes into the cluster, as well as on the total area of the cluster S_n, $Q_n = \sqrt{\frac{nS_n}{S_1}}Q_1$. We take S_1 constant and equal to a disc of radius $r_\perp \simeq 0.2$ fm. S_n corresponds to the total area occupied by n discs[1]. Knowing the colour

[1] Notice that if the strings don't overlap, $S_n = nS_1$ and $Q_n = nQ_1$, so the strings act independently. On the contrary, if they fully overlap, $S_n = S_1$ and $Q_n = \sqrt{n}Q_1$.

charge Q_n, one can compute the multiplicity μ_n and the mean transverse momentum $< p_T^2 >_n$ of the particles produced by a cluster of n strings. One finds [5]

$$\mu_n = \sqrt{\frac{nS_n}{S_1}}\mu_1 \ , \qquad < p_T^2 >_n = \sqrt{\frac{nS_1}{S_n}} < p_T^2 >_1 \tag{12}$$

where μ_1 and $< p_T^2 >_1$ are the mean multiplicity and mean p_T^2 of particles produced by a simple string. In the saturation limit, i. e. all the strings overlap into a single cluster that occupies the whole interaction area, one gets the following scaling law that relates the mean transverse momentum and the multiplicity per unit rapidity and unit transverse area:

$$< p_T^2 >_{AA} = \frac{S_1}{S_{AA}} \frac{< p_T^2 >_1}{\mu_1} \mu_{AA} \ . \tag{13}$$

This scaling relation is similar to the one obtained in the framework of the CGC when the initial gluon density saturates.

Moreover, in the limit of high density $\eta = N_s S_1/S_{AA}$, one obtains

$$< \frac{nS_1}{S_n} > = \frac{\eta}{1 - \exp(-\eta)} \equiv \frac{1}{F(\eta)^2} \tag{14}$$

and the equations (12) transform into the analytic ones [6]

$$\mu = N_s F(\eta)\mu_1 \ , \qquad < p_T^2 > = \frac{1}{F(\eta)} < p_T^2 >_1 \tag{15}$$

where μ and $< p_T^2 >$ are the total multiplicity and mean momentum, and N_s is the total number of strings created in the collision.

In order to study the transverse momentum distribution, one needs the distribution $f(x, m_T)$ for each string or cluster, and the cluster size distribution $W(x)$. For $f(x, m_T)$ we assume the Schwinger formula, $f(x, m_T) = \exp(-m_T^2 x F(\eta))$, used also for the fragmentation of a Lund string. In this formula x is related to the string tension, or equivalently to the mean transverse size of the string. The weight function $W(x)$ obeys a gamma distribution [6]. Assuming that a cluster behaves similarly to a single string but with different string tension, that depends on the number of strings that come into the cluster, we can write for the total p_T distribution

$$f(m_T) = \int W(x) \ f(x, m_T) \, , \qquad W(x) = \frac{\gamma}{\Gamma(k)}(\gamma x)^{k-1} \ \exp(-\gamma x) \ . \tag{16}$$

Performing the integral in eq. (16) we obtain

$$\frac{\mathrm{d}N}{\mathrm{d}p_T^2 \mathrm{d}y} = \frac{\mathrm{d}N}{\mathrm{d}y} \frac{k-1}{\gamma} F(\eta) \frac{1}{(1 + \frac{F(\eta) \ p_T^2}{\gamma})^k} \ , \tag{17}$$

where k can be obtained from the fluctuations in multiplicy.

3. SOME SIMILARITIES: The results from string clustering and from the CGC

To finish, let us remember some of the similarities of the explained approaches:

• In the clustering approach, when taking the saturation limit –all the strings overlap into a single cluster that occupies the whole nuclear overlap area–, one finds that the particle multiplicity of a central collision, μ_{AA}, behaves as $\mu_{AA} = \mu_n = \sqrt{\frac{nS_n}{S_1}}\mu_1 = \sqrt{\frac{N_s S_{AA}}{S_1}}\mu_1$. Taking into account that the number of strings produced in the nuclear collision, N_s, is proportional to the number of inelastic nucleon-nucleon collisions, $N_{coll} \sim A^{4/3}$, and S_{AA} corresponds to the nuclear overlap area, $S_{AA} \sim A^{2/3}$, we obtain a multiplicity that scales with the number of participants A. This coincides with the multiplicity obtained in first approximation – without evolution – in the framework of the CGC (eq. (10)).

• In the CGC, there is a relation between the mean transverse momentum and the multiplicity, which is developed in eq. (11). This relation shows that at saturation the mean transverse momentum should scale with the multiplicity per unit rapidity and unit transverse area. This coincides with the proportionality relation obtained in the clustering model (eq. (13)).

• In both approaches, the initial state interactions –gluon saturation in the CGC or clustering of strings– produce a suppression of high p_T and multiplicities. On the contrary, in the framework of the jet quenching phenomena, the energy loss of the jet with a hot and dense medium produces additional soft gluons that would fragment into hadrons increasing the multiplicities, unless strong shadowing occurs in the gluon structure functions.

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